to “pop” during takeoff in an airplane. Changes in pressure can also cause scuba divers to suffer a sometimes fatal disorder known as the “bends,” which occurs when nitrogen dissolved in the water of the body at extreme depths returns to a gaseous state in the body as the diver surfaces. Pressure lies at the heart of the phenomena called buoyancy, which causes hot air balloons to rise and ships to float. Before we can fully understand the role that pressure plays in these phenomena, we need to discuss the states of matter and the concept of density.

14.1 | Fluids, Density, and Pressure

Learning Objectives

By the end of this section, you will be able to:

- State the different phases of matter
- Describe the characteristics of the phases of matter at the molecular or atomic level
- Distinguish between compressible and incompressible materials
- Define density and its related SI units
- Compare and contrast the densities of various substances
- Define pressure and its related SI units
- Explain the relationship between pressure and force
- Calculate force given pressure and area

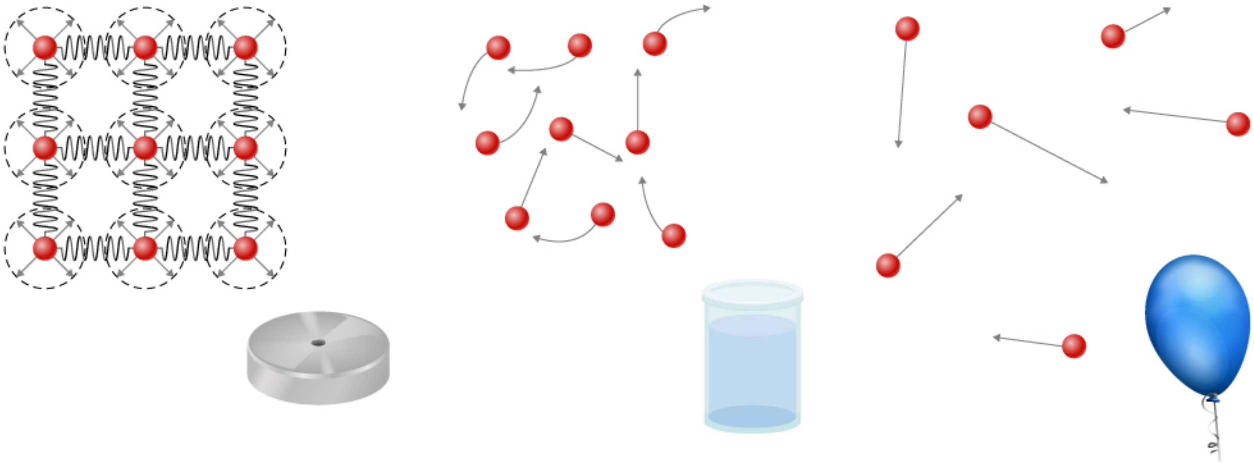
Matter most commonly exists as a solid, liquid, or gas; these states are known as the three common phases of matter. We will look at each of these phases in detail in this section.

Characteristics of Solids

Solids are rigid and have specific shapes and definite volumes. The atoms or molecules in a solid are in close proximity to each other, and there is a significant force between these molecules. Solids will take a form determined by the nature of these forces between the molecules. Although true solids are not incompressible, it nevertheless requires a large force to change the shape of a solid. In some cases, the force between molecules can cause the molecules to organize into a lattice as shown in **Figure 14.2**. The structure of this three-dimensional lattice is represented as molecules connected by rigid bonds (modeled as stiff springs), which allow limited freedom for movement. Even a large force produces only small displacements in the atoms or molecules of the lattice, and the solid maintains its shape. Solids also resist shearing forces. (Shearing forces are forces applied tangentially to a surface, as described in **Static Equilibrium and Elasticity**.)

Characteristics of Fluids

Liquids and gases are considered to be **fluids** because they yield to shearing forces, whereas solids resist them. Like solids, the molecules in a liquid are bonded to neighboring molecules, but possess many fewer of these bonds. The molecules in a liquid are not locked in place and can move with respect to each other. The distance between molecules is similar to the distances in a solid, and so liquids have definite volumes, but the shape of a liquid changes, depending on the shape of its container. Gases are not bonded to neighboring atoms and can have large separations between molecules. Gases have neither specific shapes nor definite volumes, since their molecules move to fill the container in which they are held (**Figure 14.2**).



(a) (b) (c)

Figure 14.2 (a) Atoms in a solid are always in close contact with neighboring atoms, held in place by forces represented here by springs. (b) Atoms in a liquid are also in close contact but can slide over one another. Forces between the atoms strongly resist attempts to compress the atoms. (c) Atoms in a gas move about freely and are separated by large distances. A gas must be held in a closed container to prevent it from expanding freely and escaping.

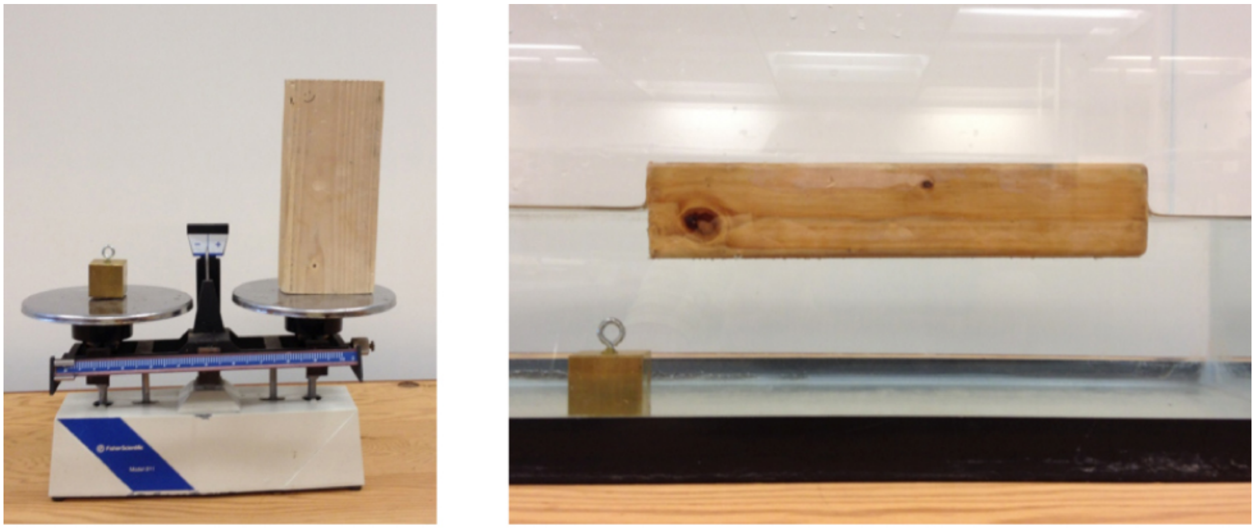
Liquids deform easily when stressed and do not spring back to their original shape once a force is removed. This occurs because the atoms or molecules in a liquid are free to slide about and change neighbors. That is, liquids flow (so they are a type of fluid), with the molecules held together by mutual attraction. When a liquid is placed in a container with no lid, it remains in the container. Because the atoms are closely packed, liquids, like solids, resist compression; an extremely large force is necessary to change the volume of a liquid.

In contrast, atoms in gases are separated by large distances, and the forces between atoms in a gas are therefore very weak, except when the atoms collide with one another. This makes gases relatively easy to compress and allows them to flow (which makes them fluids). When placed in an open container, gases, unlike liquids, will escape.

In this chapter, we generally refer to both gases and liquids simply as fluids, making a distinction between them only when they behave differently. There exists one other phase of matter, plasma, which exists at very high temperatures. At high temperatures, molecules may disassociate into atoms, and atoms disassociate into electrons (with negative charges) and protons (with positive charges), forming a plasma. Plasma will not be discussed in depth in this chapter because plasma has very different properties from the three other common phases of matter, discussed in this chapter, due to the strong electrical forces between the charges.

Density

Suppose a block of brass and a block of wood have exactly the same mass. If both blocks are dropped in a tank of water, why does the wood float and the brass sink (Figure 14.3)? This occurs because the brass has a greater density than water, whereas the wood has a lower density than water.



(a) (b)

Figure 14.3 (a) A block of brass and a block of wood both have the same weight and mass, but the block of wood has a much greater volume. (b) When placed in a fish tank filled with water, the cube of brass sinks and the block of wood floats. (The block of wood is the same in both pictures; it was turned on its side to fit on the scale.) (credit: modification of works by Joseph J. Trout, Stockton University)

Density is an important characteristic of substances. It is crucial, for example, in determining whether an object sinks or floats in a fluid.

Density

The average density of a substance or object is defined as its mass per unit volume,

$$\rho = \frac{m}{V} \tag{14.1}$$

where the Greek letter ρ (rho) is the symbol for density, m is the mass, and V is the volume.

The SI unit of density is kg/m^3. **Table 14.1** lists some representative values. The cgs unit of density is the gram per cubic centimeter, g/cm^3, where

$$1 \text{ g/cm}^3 = 1000 \text{ kg/m}^3.$$

The metric system was originally devised so that water would have a density of 1 g/cm^3, equivalent to 10^3 kg/m^3. Thus, the basic mass unit, the kilogram, was first devised to be the mass of 1000 mL of water, which has a volume of 1000 cm^3.

Solids ($0.0°\text{C}$)		Liquids ($0.0°\text{C}$)		Gases ($0.0°\text{C}$, 101.3 kPa)	
Substance	$\rho(\text{kg/m}^3)$	**Substance**	$\rho(\text{kg/m}^3)$	**Substance**	$\rho(\text{kg/m}^3)$
Aluminum	2.70×10^3	Benzene	8.79×10^2	Air	1.29×10^0
Bone	1.90×10^3	Blood	1.05×10^3	Carbon dioxide	1.98×10^0
Brass	8.44×10^3	Ethyl alcohol	8.06×10^2	Carbon monoxide	1.25×10^0

Table 14.1 Densities of Some Common Substances

Solids ($0.0°C$)		Liquids ($0.0°C$)		Gases ($0.0°C,$ 101.3 kPa)	
Concrete	2.40×10^3	Gasoline	6.80×10^2	Helium	1.80×10^{-1}
Copper	8.92×10^3	Glycerin	1.26×10^3	Hydrogen	9.00×10^{-2}
Cork	2.40×10^2	Mercury	1.36×10^4	Methane	7.20×10^{-2}
Earth's crust	3.30×10^3	Olive oil	9.20×10^2	Nitrogen	1.25×10^0
Glass	2.60×10^3			Nitrous oxide	1.98×10^0
Gold	1.93×10^4			Oxygen	1.43×10^0
Granite	2.70×10^3				
Iron	7.86×10^3				
Lead	1.13×10^4				
Oak	7.10×10^2				
Pine	3.73×10^2				
Platinum	2.14×10^4				
Polystyrene	1.00×10^2				
Tungsten	1.93×10^4				
Uranium	1.87×10^3				

Table 14.1 Densities of Some Common Substances

As you can see by examining **Table 14.1**, the density of an object may help identify its composition. The density of gold, for example, is about 2.5 times the density of iron, which is about 2.5 times the density of aluminum. Density also reveals something about the phase of the matter and its substructure. Notice that the densities of liquids and solids are roughly comparable, consistent with the fact that their atoms are in close contact. The densities of gases are much less than those of liquids and solids, because the atoms in gases are separated by large amounts of empty space. The gases are displayed for a standard temperature of $0.0°C$ and a standard pressure of 101.3 kPa, and there is a strong dependence of the densities on temperature and pressure. The densities of the solids and liquids displayed are given for the standard temperature of $0.0°C$ and the densities of solids and liquids depend on the temperature. The density of solids and liquids normally increase with decreasing temperature.

Table 14.2 shows the density of water in various phases and temperature. The density of water increases with decreasing temperature, reaching a maximum at $4.0°C,$ and then decreases as the temperature falls below $4.0°C$. This behavior of the density of water explains why ice forms at the top of a body of water.

Substance	$\rho(\text{kg/m}^3)$
Ice $(0°C)$	9.17×10^2
Water $(0°C)$	9.998×10^2
Water $(4°C)$	1.000×10^3

Table 14.2 Densities of Water

Substance	$\rho(\mathbf{kg/m^3})$
Water $(20°C)$	9.982×10^2
Water $(100°C)$	9.584×10^2
Steam $(100°C, 101.3\ kPa)$	1.670×10^2
Sea water $(0°C)$	1.030×10^3

Table 14.2 Densities of Water

The density of a substance is not necessarily constant throughout the volume of a substance. If the density is constant throughout a substance, the substance is said to be a homogeneous substance. A solid iron bar is an example of a homogeneous substance. The density is constant throughout, and the density of any sample of the substance is the same as its average density. If the density of a substance were not constant, the substance is said to be a heterogeneous substance. A chunk of Swiss cheese is an example of a heterogeneous material containing both the solid cheese and gas-filled voids. The density at a specific location within a heterogeneous material is called *local density*, and is given as a function of location, $\rho = \rho(x,\ y,\ z)$ (**Figure 14.4**).

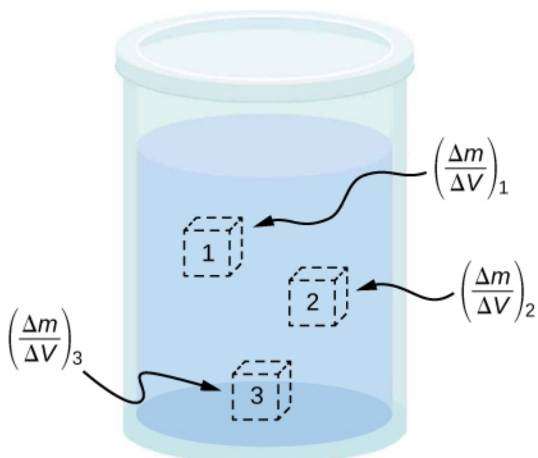


Figure 14.4 Density may vary throughout a heterogeneous mixture. Local density at a point is obtained from dividing mass by volume in a small volume around a given point.

Local density can be obtained by a limiting process, based on the average density in a small volume around the point in question, taking the limit where the size of the volume approaches zero,

$$\rho = \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V} \tag{14.2}$$

where ρ is the density, m is the mass, and V is the volume.

Since gases are free to expand and contract, the densities of the gases vary considerably with temperature, whereas the densities of liquids vary little with temperature. Therefore, the densities of liquids are often treated as constant, with the density equal to the average density.

Density is a dimensional property; therefore, when comparing the densities of two substances, the units must be taken into consideration. For this reason, a more convenient, dimensionless quantity called the **specific gravity** is often used to compare densities. Specific gravity is defined as the ratio of the density of the material to the density of water at $4.0\ °C$ and one atmosphere of pressure, which is $1000\ kg/m^3$:

$$\text{Specific} \quad \text{ravity} = \frac{\text{Density of material}}{\text{Density of water}}.$$

The comparison uses water because the density of water is 1 g/cm^3, which was originally used to define the kilogram. Specific gravity, being dimensionless, provides a ready comparison among materials without having to worry about the unit of density. For instance, the density of aluminum is 2.7 in g/cm^3 (2700 in kg/m^3), but its specific gravity is 2.7, regardless of the unit of density. Specific gravity is a particularly useful quantity with regard to buoyancy, which we will discuss later in this chapter.

Pressure

You have no doubt heard the word 'pressure' used in relation to blood (high or low blood pressure) and in relation to weather (high- and low-pressure weather systems). These are only two of many examples of pressure in fluids. (Recall that we introduced the idea of pressure in **Static Equilibrium and Elasticity**, in the context of bulk stress and strain.)

Pressure

Pressure (p) is defined as the normal force F per unit area A over which the force is applied, or

$$p = \frac{F}{A}. \tag{14.3}$$

To define the pressure at a specific point, the pressure is defined as the force dF exerted by a fluid over an infinitesimal element of area dA containing the point, resulting in $p = \frac{dF}{dA}$.

A given force can have a significantly different effect, depending on the area over which the force is exerted. For instance, a force applied to an area of 1 mm^2 has a pressure that is 100 times as great as the same force applied to an area of 1 cm^2. That is why a sharp needle is able to poke through skin when a small force is exerted, but applying the same force with a finger does not puncture the skin (**Figure 14.5**).

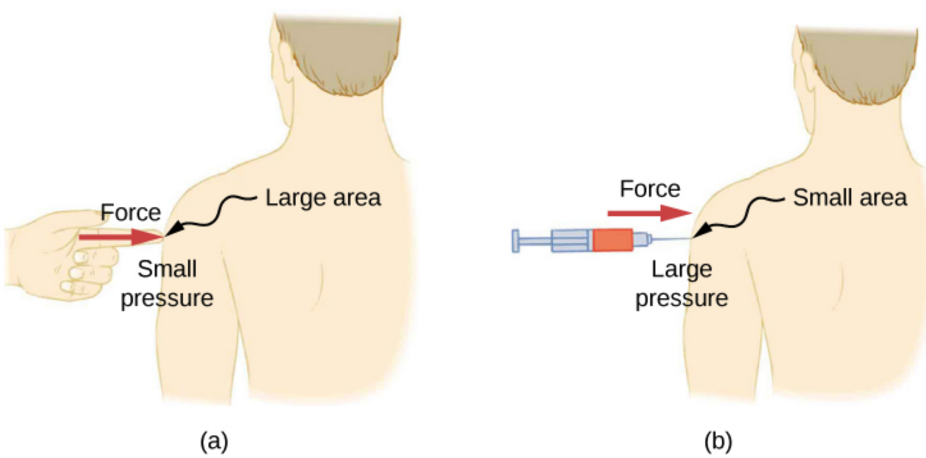


Figure 14.5 (a) A person being poked with a finger might be irritated, but the force has little lasting effect. (b) In contrast, the same force applied to an area the size of the sharp end of a needle is enough to break the skin.

Note that although force is a vector, pressure is a scalar. Pressure is a scalar quantity because it is defined to be proportional to the magnitude of the force acting perpendicular to the surface area. The SI unit for pressure is the *pascal* (Pa), named after the French mathematician and physicist Blaise Pascal (1623–1662), where

$$1 \text{ Pa} = 1 \text{ N/m}^2.$$

Several other units are used for pressure, which we discuss later in the chapter.

Variation of pressure with depth in a fluid of constant density

Pressure is defined for all states of matter, but it is particularly important when discussing fluids. An important characteristic of fluids is that there is no significant resistance to the component of a force applied parallel to the surface of a fluid. The molecules of the fluid simply flow to accommodate the horizontal force. A force applied perpendicular to the surface compresses or expands the fluid. If you try to compress a fluid, you find that a reaction force develops at each point inside the fluid in the outward direction, balancing the force applied on the molecules at the boundary.

Consider a fluid of constant density as shown in **Figure 14.6**. The pressure at the bottom of the container is due to the pressure of the atmosphere (p_0) plus the pressure due to the weight of the fluid. The pressure due to the fluid is equal to the weight of the fluid divided by the area. The weight of the fluid is equal to its mass times the acceleration due to gravity.

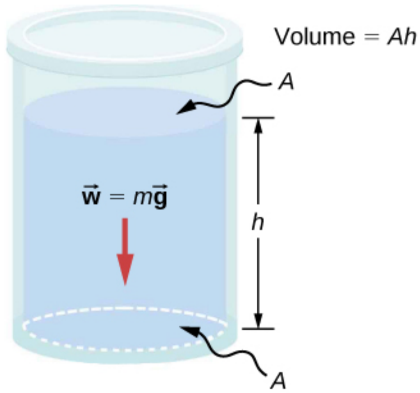


Figure 14.6 The bottom of this container supports the entire weight of the fluid in it. The vertical sides cannot exert an upward force on the fluid (since it cannot withstand a shearing force), so the bottom must support it all.

Since the density is constant, the weight can be calculated using the density:

$$w = mg = \rho V g = \rho A h g.$$

The pressure at the bottom of the container is therefore equal to atmospheric pressure added to the weight of the fluid divided by the area:

$$p = p_0 + \frac{\rho A h g}{A} = p_0 + \rho h g.$$

This equation is only good for pressure at a depth for a fluid of constant density.

Pressure at a Depth for a Fluid of Constant Density

The pressure at a depth in a fluid of constant density is equal to the pressure of the atmosphere plus the pressure due to the weight of the fluid, or

$$p = p_0 + \rho h g, \qquad (14.4)$$

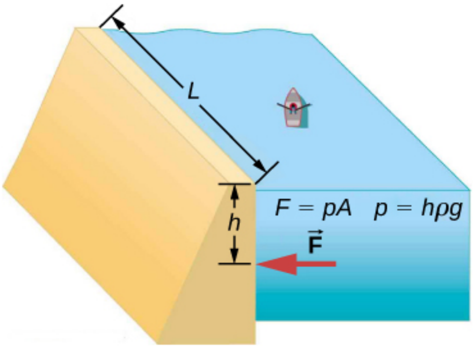
Where p is the pressure at a particular depth, p_0 is the pressure of the atmosphere, ρ is the density of the fluid, g is the acceleration due to gravity, and h is the depth.

Figure 14.7 The Three Gorges Dam, erected on the Yangtze River in central China in 2008, created a massive reservoir that displaced more than one million people. (credit: modification of work by “Le Grand Portage”/Flickr)

Example 14.1

What Force Must a Dam Withstand?

Consider the pressure and force acting on the dam retaining a reservoir of water (**Figure 14.7**). Suppose the dam is 500-m wide and the water is 80.0-m deep at the dam, as illustrated below. (a) What is the average pressure on the dam due to the water? (b) Calculate the force exerted against the dam.



The average pressure p due to the weight of the water is the pressure at the average depth h of 40.0 m, since pressure increases linearly with depth. The force exerted on the dam by the water is the average pressure times the area of contact, $F = pA$.

solution

a. The average pressure due to the weight of a fluid is

$$p = h\rho g. \quad (14.5)$$

Entering the density of water from **Table 14.1** and taking h to be the average depth of 40.0 m, we obtain

$$\begin{aligned} p &= (40.0\,\text{m})\left(10^3\,\frac{\text{kg}}{\text{m}^3}\right)\left(9.80\frac{\text{m}}{\text{s}^2}\right) \\ &= 3.92 \times 10^5\,\frac{\text{N}}{\text{m}^2} = 392\,\text{kPa}. \end{aligned}$$

b. We have already found the value for p. The area of the dam is

$$A = 80.0\,\text{m} \times 500\,\text{m} = 4.00 \times 10^4\,\text{m}^2,$$

so that

$$F = (3.92 \times 10^5 \text{ N/m}^2)(4.00 \times 10^4 \text{ m}^2)$$
$$= 1.57 \times 10^{10} \text{ N}.$$

Significance

Although this force seems large, it is small compared with the 1.96×10^{13} N weight of the water in the reservoir. In fact, it is only 0.0800% of the weight.

14.1 Check Your Understanding If the reservoir in **Example 14.1** covered twice the area, but was kept to the same depth, would the dam need to be redesigned?

Pressure in a static fluid in a uniform gravitational field

A *static fluid* is a fluid that is not in motion. At any point within a static fluid, the pressure on all sides must be equal—otherwise, the fluid at that point would react to a net force and accelerate.

The pressure at any point in a static fluid depends only on the depth at that point. As discussed, pressure in a fluid near Earth varies with depth due to the weight of fluid above a particular level. In the above examples, we assumed density to be constant and the average density of the fluid to be a good representation of the density. This is a reasonable approximation for liquids like water, where large forces are required to compress the liquid or change the volume. In a swimming pool, for example, the density is approximately constant, and the water at the bottom is compressed very little by the weight of the water on top. Traveling up in the atmosphere is quite a different situation, however. The density of the air begins to change significantly just a short distance above Earth's surface.

To derive a formula for the variation of pressure with depth in a tank containing a fluid of density ρ on the surface of Earth, we must start with the assumption that the density of the fluid is not constant. Fluid located at deeper levels is subjected to more force than fluid nearer to the surface due to the weight of the fluid above it. Therefore, the pressure calculated at a given depth is different than the pressure calculated using a constant density.

Imagine a thin element of fluid at a depth h, as shown in **Figure 14.8**. Let the element have a cross-sectional area A and height Δy. The forces acting upon the element are due to the pressures $p(y)$ above and $p(y + \Delta y)$ below it. The weight of the element itself is also shown in the free-body diagram.

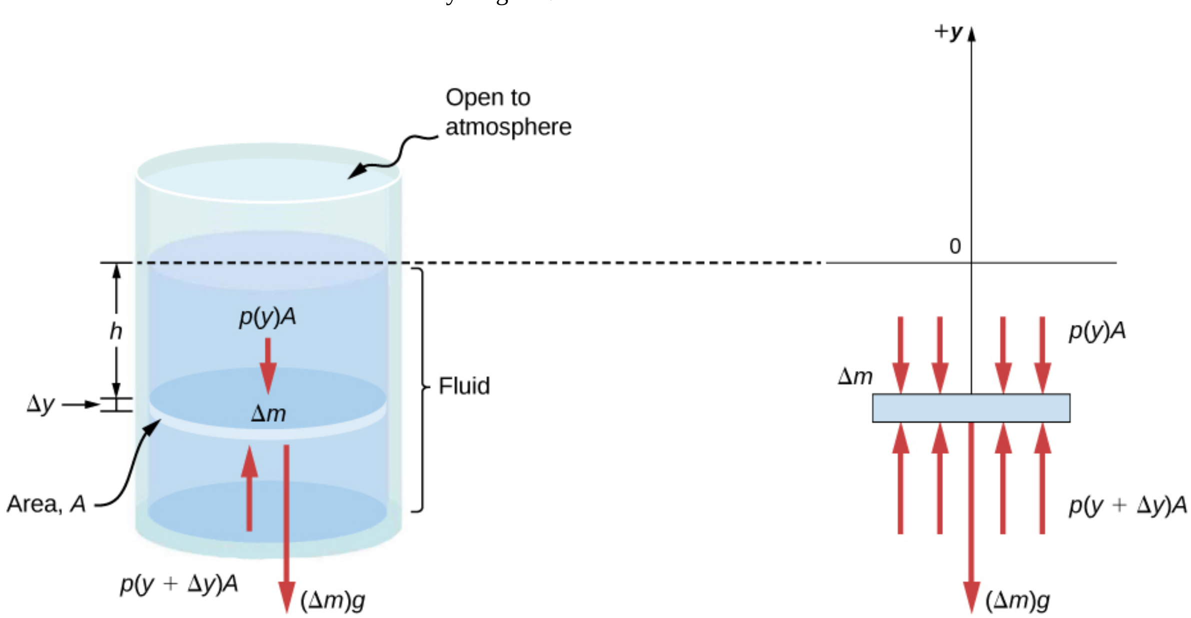


Figure 14.8 Forces on a mass element inside a fluid. The weight of the element itself is shown in the free-body diagram.

Since the element of fluid between y and $y + \Delta y$ is not accelerating, the forces are balanced. Using a Cartesian y-axis oriented up, we find the following equation for the *y*-component:

$$p(y+\Delta y)A - p(y)A - g\Delta m = 0(\Delta y > 0). \quad (14.6)$$

Note that if the element had a non-zero *y*-component of acceleration, the right-hand side would not be zero but would instead be the mass times the *y*-acceleration. The mass of the element can be written in terms of the density of the fluid and the volume of the elements:

$$\Delta m = |\rho A \Delta y| = -\rho A \Delta y \quad (\Delta y > 0).$$

Putting this expression for Δm into **Equation 14.6** and then dividing both sides by $A\Delta y$, we find

$$\frac{p(y+\Delta y) - p(y)}{\Delta y} = -\rho g. \quad (14.7)$$

Taking the limit of the infinitesimally thin element $\Delta y \to 0$, we obtain the following differential equation, which gives the variation of pressure in a fluid:

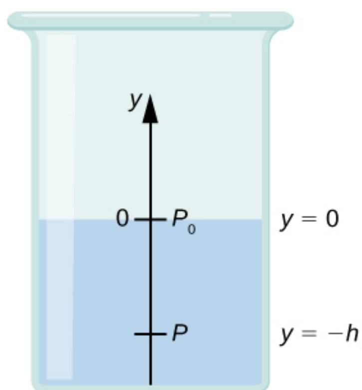
$$\frac{dp}{dy} = -\rho g. \quad (14.8)$$

This equation tells us that the rate of change of pressure in a fluid is proportional to the density of the fluid. The solution of this equation depends upon whether the density ρ is constant or changes with depth; that is, the function $\rho(y)$.

If the range of the depth being analyzed is not too great, we can assume the density to be constant. But if the range of depth is large enough for the density to vary appreciably, such as in the case of the atmosphere, there is significant change in density with depth. In that case, we cannot use the approximation of a constant density.

Pressure in a fluid with a constant density

Let's use **Equation 14.9** to work out a formula for the pressure at a depth *h* from the surface in a tank of a liquid such as water, where the density of the liquid can be taken to be constant.



We need to integrate **Equation 14.9** from $y = 0$, where the pressure is atmospheric pressure (p_0), to $y = -h$, the *y*-coordinate of the depth:

$$\begin{aligned} \int_{p_0}^{p} dp &= -\int_0^{-h} \rho g dy \\ p - p_0 &= \rho g h \\ p &= p_0 + \rho g h. \end{aligned} \quad (14.9)$$

Hence, pressure at a depth of fluid on the surface of Earth is equal to the atmospheric pressure plus ρgh if the density of the fluid is constant over the height, as we found previously.

Note that the pressure in a fluid depends only on the depth from the surface and not on the shape of the container. Thus, in a container where a fluid can freely move in various parts, the liquid stays at the same level in every part, regardless of the shape, as shown in **Figure 14.9**.

Figure 14.9 If a fluid can flow freely between parts of a container, it rises to the same height in each part. In the container pictured, the pressure at the bottom of each column is the same; if it were not the same, the fluid would flow until the pressures became equal.

Variation of atmospheric pressure with height

The change in atmospheric pressure with height is of particular interest. Assuming the temperature of air to be constant, and that the ideal gas law of thermodynamics describes the atmosphere to a good approximation, we can find the variation of atmospheric pressure with height, when the temperature is constant. (We discuss the ideal gas law in a later chapter, but we assume you have some familiarity with it from high school and chemistry.) Let $p(y)$ be the atmospheric pressure at height y. The density ρ at y, the temperature T in the Kelvin scale (K), and the mass m of a molecule of air are related to the absolute pressure by the ideal gas law, in the form

$$p = \rho \frac{k_B T}{m} \text{(atmosphere)}, \tag{14.10}$$

where k_B is Boltzmann's constant, which has a value of 1.38×10^{-23} J/K.

You may have encountered the ideal gas law in the form $pV = nRT$, where n is the number of moles and R is the gas constant. Here, the same law has been written in a different form, using the density ρ instead of volume V. Therefore, if pressure p changes with height, so does the density ρ. Using density from the ideal gas law, the rate of variation of pressure with height is given as

$$\frac{dp}{dy} = -p\left(\frac{mg}{k_B T}\right),$$

where constant quantities have been collected inside the parentheses. Replacing these constants with a single symbol α, the equation looks much simpler:

$$\begin{aligned}
\frac{dp}{dy} &= -\alpha p \\
\frac{dp}{p} &= -\alpha dy \\
\int_{p_0}^{p(y)} \frac{dp}{p} &= \int_0^y -\alpha dy \\
[\ln(p)]_{p_0}^{p(y)} &= [-\alpha y]_0^y \\
\ln(p) - \ln(p_0) &= -\alpha y \\
\ln\left(\frac{p}{p_0}\right) &= -\alpha y
\end{aligned}$$

This gives the solution

$$p(y) = p_0 \exp(-\alpha y).$$

Thus, atmospheric pressure drops exponentially with height, since the y-axis is pointed up from the ground and y has positive values in the atmosphere above sea level. The pressure drops by a factor of $\frac{1}{e}$ when the height is $\frac{1}{\alpha}$, which gives us a physical interpretation for α: The constant $\frac{1}{\alpha}$ is a length scale that characterizes how pressure varies with height and is often referred to as the pressure scale height.

We can obtain an approximate value of α by using the mass of a nitrogen molecule as a proxy for an air molecule. At temperature $27\ °\text{C}$, or 300 K, we find

$$\alpha = -\frac{mg}{k_B T} = \frac{4.8 \times 10^{-26}\ \text{kg} \times 9.81\ \text{m/s}^2}{1.38 \times 10^{-23}\ \text{J/K} \times 300\ \text{K}} = \frac{1}{8800\ \text{m}}.$$

Therefore, for every 8800 meters, the air pressure drops by a factor $1/e$, or approximately one-third of its value. This gives us only a rough estimate of the actual situation, since we have assumed both a constant temperature and a constant g over such great distances from Earth, neither of which is correct in reality.

Direction of pressure in a fluid

Fluid pressure has no direction, being a scalar quantity, whereas the forces due to pressure have well-defined directions: They are always exerted perpendicular to any surface. The reason is that fluids cannot withstand or exert shearing forces. Thus, in a static fluid enclosed in a tank, the force exerted on the walls of the tank is exerted perpendicular to the inside surface. Likewise, pressure is exerted perpendicular to the surfaces of any object within the fluid. **Figure 14.10** illustrates the pressure exerted by air on the walls of a tire and by water on the body of a swimmer.

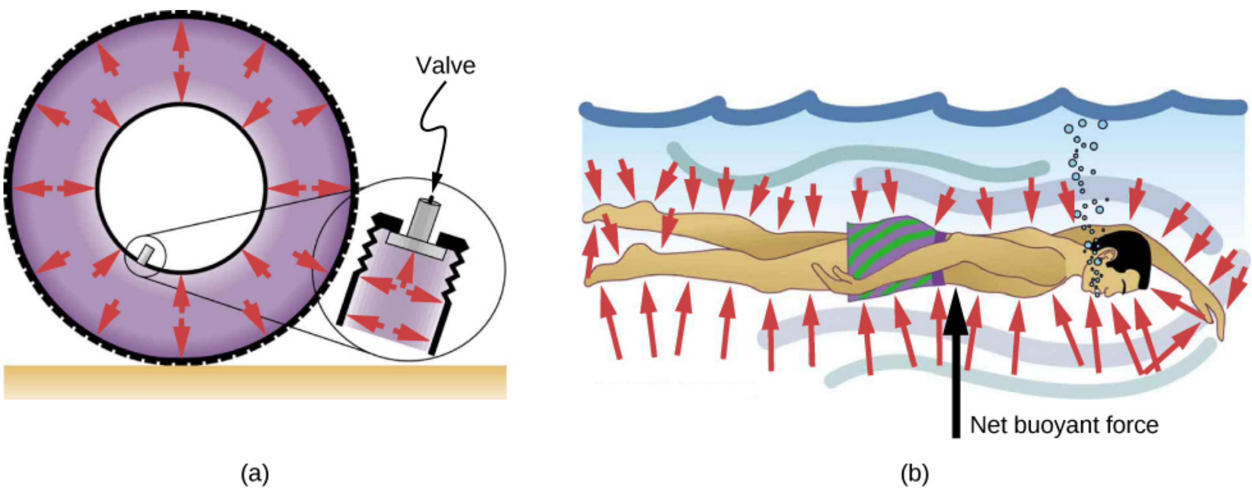


Figure 14.10 (a) Pressure inside this tire exerts forces perpendicular to all surfaces it contacts. The arrows represent directions and magnitudes of the forces exerted at various points. (b) Pressure is exerted perpendicular to all sides of this swimmer, since the water would flow into the space he occupies if he were not there. The arrows represent the directions and magnitudes of the forces exerted at various points on the swimmer. Note that the forces are larger underneath, due to greater depth, giving a net upward or buoyant force. The net vertical force on the swimmer is equal to the sum of the buoyant force and the weight of the swimmer.

14.2 | Measuring Pressure

Learning Objectives

By the end of this section, you will be able to:

- Define gauge pressure and absolute pressure
- Explain various methods for measuring pressure
- Understand the working of open-tube barometers
- Describe in detail how manometers and barometers operate

In the preceding section, we derived a formula for calculating the variation in pressure for a fluid in hydrostatic equilibrium. As it turns out, this is a very useful calculation. Measurements of pressure are important in daily life as well as in science